"Inverted" Fisher's Model at Coexistence: Another Approach to Cluster Analysis

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Fisher's droplet model [1] has had sweeping success in describing nuclear multifragmentation yields and construction of the nuclear liquid-vapour phase diagram [2]. However it suffers from an inability to account for excluded volume effects in dense cluster environments. At high density, formation of clusters is obstructed by the presence of other clusters. Such effects lead to strict cluster non-independence and failure of Fisher's model in its original form.

We recently came up with a modification of the way Fisher's model is used to analyze gas properties in the two-phase region combining ideas from Stillinger's cluster model [3] and the fact that the calculation of the internal energy of the system does not depend on the assumption of an ideal gas of clusters. Thus the internal energy of the system per unit volume can be calculated from the cluster concentrations as

$$u = \sum_a \bar{s}_a n(a,T) = T\frac{\partial p}{\partial T} - p \tag{1}$$

where $n(a,T)$ is the concentration of clusters of size a, $\bar{s}_a$ is their average surface, $T = 1/\beta$ is the temperature, and p is the pressure.

According to Stillinger, formation of a cluster is hindered by the presence of other clusters, which need to be pushed aside to form a cavity. This requires additional work. Thus the cluster concentrations should contain the cavity formation probability, which is pressure (p) and density (ρ) dependent. We can modify Fisher's droplet concentrations at coexistence to include excluded volume effects as follows:

$$n(a,T,p,\rho) = g(\bar{s}_a)\exp[-c\beta\bar{s}_a]\exp[-\beta W(a,T,p,\rho)], \tag{2}$$

where c is the surface energy coefficient, and to the lowest order the energy to form a cavity of volume v is

$$W(a,T,p,\rho) \approx pv(a) - T\ln(1-\rho), \tag{3}$$

which follows from the work of Reiss, Frisch, Lebowitz [4], and Stillinger. Recalling the definition of the density, and putting all the above ideas together, a system of equations can be written

$$\begin{aligned} \sum_a \bar{s}_a n(a,T,p,\rho) &= T\frac{\partial p}{\partial T} - p \\ \sum_a a n(a,T,p,\rho) &= \rho \end{aligned} \tag{4}$$

which defines the properties of the two-phase region within the validity of Eq. (3) and Fisher's assumptions about $\bar{s}_a$ and $g(\bar{s}_a)$. The solution of Eqs. (4) with initial condition $p(T=0) = 0$ can be combined with a χ^2-minimization procedure to analyze experimental cluster distributions and obtain the phase diagram.

As a test we turned to a well-known model of phase transition that can generate cluster distributions at coexistence: the zero-field two-dimensional Ising model. We used geometric configurational clusters that are defined only by the nearest neighbour condition. We fitted cluster concentrations from a simulation using the above technique, and when the fitting successfully converged we compared the pressure with the exactly known result from the Onsager solution [5] using the equivalence relations between the Ising model and the lattice gas established by Lee and Yang [6] (see Fig. 1). The agreement suggests that the method might be used to look for excluded volume effects in nuclear clusters, as well as provide an alternative technique to verify the nuclear liquid-vapour phase diagram. The work is underway. Details of the procedure will appear in [7].

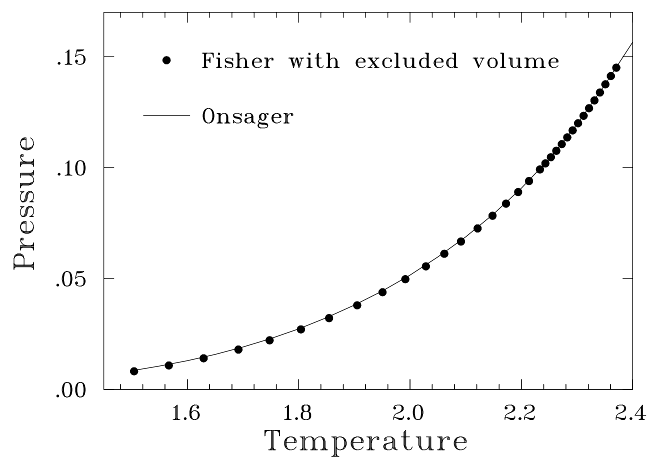

FIG. 1: The two-dimensional Ising pressure obtained from the analysis of geometric clusters as compared to the exact Onsager result.

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